Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70772

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth/Fifth/Sixth/Seventh Semester

Civil Engineering

MA 6459 — NUMERICAL METHODS

(Common to Aeronautical Engineering/Agriculture Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical and Automation Engineering/Petrochemical Engineering/Production Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Handloom and Textile Technology/Petrochemical Technology/Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology)

(Regulation 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is the criterion for the convergence of Newton-Raphson method?

2. Give two direct methods to solve a system of linear equations.

3. Given $y_0 = 3$, $y_1 = 12$, $y_2 = 81$, $y_3 = 200$, $y_4 = 100$. Find $\Delta^4 y_0$.

4. Distinguish between Newton divided difference interpolation and Lagrange's interpolation.

5. Write down the general quadrature formula for equidistance ordinates.

6. Write down the forward difference formulae to compute the first two derivatives at $x = x_0$.

7. Compare Single-step method and Multi-step method.

8. Write down the Milne's predictor and corrector formulas.

9. Write down the finite difference scheme for solving $y'' + x + y = 0$; $y(0) = y(1) = 0$.

10. Derive explicit finite difference scheme for $u_t = u_{xx}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the largest eigenvalue and the corresponding eigenvector of a matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. (8)

(ii) Using Gauss Jordan method find the inverse of a matrix $\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$. (8)

Or

(b) (i) Apply Gauss-Seidal method to solve the equations (8)

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35.$$

(ii) Find the root of $4x - e^x = 0$ that lies between 2 and 3 by Newton-Raphson method. (8)

12. (a) (i) Find an approximate polynomial for $f(x)$ using Lagrange's interpolation for the following data : (8)

x :	0	1	2	3
$y = f(x)$:	2	3	12	147

(ii) Find the value of y at $x = 21$ from the data given below. (8)

x :	20	23	26	29
y :	0.3420	0.3907	0.4384	0.4848

Or

(b) (i) Given the tables :

x :	5	7	11	13	17
$y = f(x)$:	150	392	1452	2366	5202

Evaluate $f(9)$ using Newton's divided difference formula. (8)

(ii) Fit a cubic spline from the given table.

x :	1	2	3
$f(x)$:	–8	–1	19

Compute $y(1.5)$ and $y(1)$ using cubic spline. (8)

13. (a) (i) Using backward difference, find $y'(2.2)$ and $y''(2.2)$ from the following table : (6)

x :	1.4	1.6	1.8	2.0	2.2
y :	4.0552	4.9530	6.0496	7.3891	9.0250

(ii) The following table gives the values of $y = \frac{1}{1+x^2}$. Take $h = 0.5$, 0.25, 0.125 and use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2} dx$. Hence deduce an approximate value of π. (10)

x :	0	0.125	0.25	0.375	0.5	0.675	0.75	0.875	1
y :	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5664	0.5

Or

(b) (i) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1 \int_0^1 \frac{dx\ dy}{1+xy}$ with $h = k = 0.25$. (8)

(ii) Evaluate $\int_0^5 \log_{10}(1+x)\,dx$ by three points Gauss quadrature formula. (8)

14. (a) (i) Using Taylor series method, compute the value of $y(0.2)$ correct to 3 decimal places from $\frac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$. (8)

(ii) Using modified Euler's method, find $y(0.1)$ and $y(0.2)$ for the given equation $\frac{dy}{dx} = x^2 + y^2$, given that $y(0) = 1$. (8)

Or

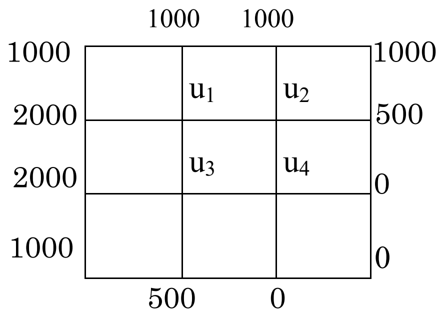
(b) (i) Find the value of $y(1.1)$ using Runge-Kutta method of 4^{th} order for the given equation $\frac{dy}{dx} = y^2 + xy;\ y(1) = 1$. (8)

(ii) Using Adam's method find $y(0.4)$ given that,

$\frac{dy}{dx} = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$. (8)

15. (a) (i) Given the values of $u(x, y)$ on the boundary of the square in fig. evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this fig. by Gauss Seidel method.

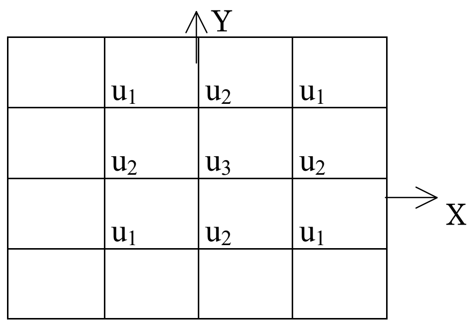
(8)



(ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x,0) = \sin \pi x$, $0 \leq x < 1$; $u(0,t) = u(1,t) = 0$ using Crank-Nicolson method. (8)

Or

(b) (i) Solve the Poisson's equation $\nabla^2 u = 8x^2 y^2$ for the square mesh of fig. with $u(x, y) = 0$ on the boundary and mesh length = 1. (8)



(ii) Evaluate the Pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = u_t(x,0) = 0$ and $u(x,0) = x^2(5 - x)$. (8)